ESCIM 2026

# BOOK OF ABSTRACTS

17th European Symposium on Computational Intelligence and Mathematics

May 10th – 13th, 2026 • Timisoara, Romania


Editors:

László T. Kóczy, Jesús Medina

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Roberto G. Aragón, Fernando Chacón-Gómez, Janusz Kacprzyk,

M. Eugenia Cornejo-Piñero, David Lobo, Francisco J. Ocaña-Alcázar, Eloísa Ramírez-Poussa

# Book of abstracts of ESCIM 2026

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Roberto G. Aragón, Fernando Chacón-Gómez, María Eugenia Cornejo,
Janusz Kacprzyk, David Lobo, Francisco J. Ocaña-Alcázar,
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# $17^{th}$ European Symposium on Computational Intelligence and Mathematics

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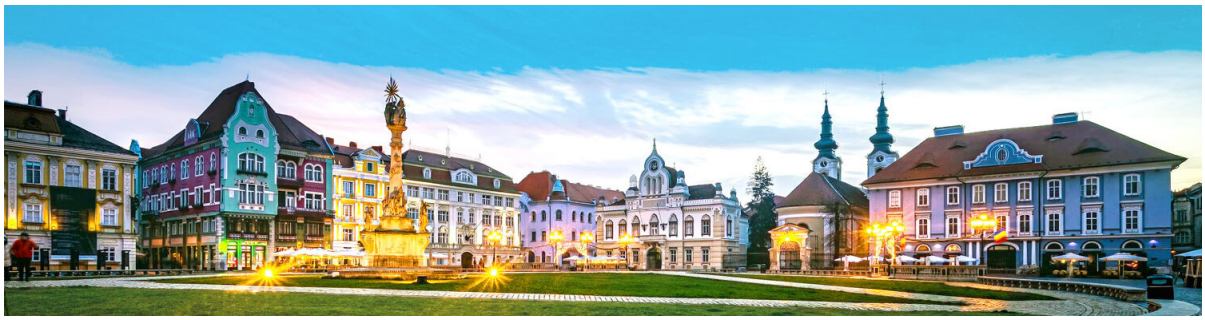


UCA Universidad de Cádiz
Departamento de Matemáticas

M·C I

| MONDAY 11th<br>Location: Continental Hotel | |
|---|---|
| 8:30 | Open Registration Desk |
| 9:00–9:30 | Inauguration |
| 9:30–10:30 | **Keynote Speaker - Yiyu Yao**<br>Title: Three-way decision: A computational intelligence perspective<br>Chairperson: Jesús Medina |
| 10:30–11:00 | Coffee break |
| 11:00–12:00 | **Special Session: Decision and optimization model applied to logistics and transport**<br>Chairperson: Julio Alberto López-Gómez |
| | *A Personality and Emotional Disregulation-based Decision system for categorizing online gambling addiction levels*<br>Jesús Serrano-Guerrero, Woorod Al-Muhtaseb, Abdullah Abdulgafer, Guadalupe Plaza, Andrés Montoro-Montarroso, Francisco Romero, José A. Olivas |
| | *Benchmarking Metaheuristic Algorithms for Revenue-Driven Timetabling in Deregulated Railway Markets*<br>David Muñoz-Valero, Juan Moreno-García, Julio Alberto López-Gómez, Enrique Adrián Villarrubia-Martín |
| | *Fairness-Aware Optimization for Train Timetabling in Liberalized Railway Markets: A Comparative Study of Equity Metrics*<br>David Muñoz-Valero, Juan Moreno-García, Julio Alberto López-Gómez, Enrique Adrián Villarrubia-Martín |
| 12:00–12:10 | Break |
| 12:10–13:10 | **Regular Session 1**<br>Chairperson: Szilvia Nagy & Boldizsár Tüű-Szabó |
| | *Deconvolutional Networks as an XAI Method for Convolutional Neural Networks in Brain MRI Classification*<br>Piotr Kowalski |
| | *A Scenario-Based Fuzzy Cognitive Map Approach to ESG Analysis in Higher Education*<br>József Hegedüs, László T. Kóczy, Miklós Hatwagner |
| | *Fuzzy Cognitive Map Modell reduction with Neural Network*<br>Barbara Sebestyén, László T. Kóczy, Miklós F. Hatwagner |
| 13:10–15:00 | Lunch |

| | |
|---|---|
| 15:00–16:00 | **Keynote Speaker - Qiang Shen**<br>Title: Harnessing AI with Limited Data: Approximate Knowledge Interpolation and Practical Applications<br>Chairperson: László T. Kóczy |
| 16:00–16:20 | Coffee break |
| 16:20–18:20 | **Special Session: Recent trends in knowledge representation and modelling. Part 1**<br>Chairperson: Ivana Štajner-Papuga |
| | *From fuzzy measures to communities: a clustering approach for interpretation*<br>Inmaculada Gutiérrez García-Pardo, Carlos I. Pérez-Sechi, Javier Castro, Daniel Gómez |
| | *Formal Concept Analysis for Evaluating the Intent Dimension of LLMs*<br>Adam Leiyi Chen, Joaquín Borrego |
| | *On the aggregation of contexts with common attributes*<br>Roberto G. Aragón, Jesús Medina, Samuel Molina-Ruiz |
| | *Interactive Fuzzy FCA Recommender System: A Visual Approach using fcaR*<br>Ángel Mora Bonilla, Domingo López-Rodríguez, Manuel Ojeda-Hernández |
| | *Definition and resolution of bipolar multi-adjoint relation equations*<br>M. Eugenia Cornejo, David Lobo, Jesús Medina |
| | *Characterizing the solvability of bipolar sup-equations in terms of the negation operator*<br>David Lobo, Pablo López-Molina |
| 20:00 | **Welcome reception** |

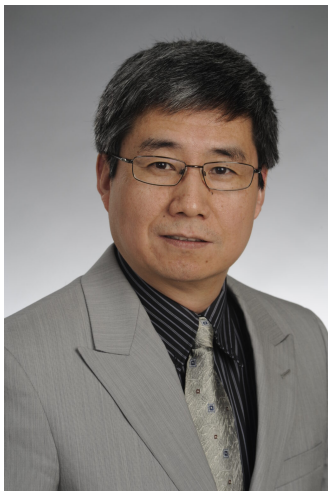
| **TUESDAY 12th**<br>**Location: Continental Hotel** | |
|---|---|
| 9:00–10:00 | **Keynote Speaker - Radu-Emul Precup**<br>Title: Model-based and Data-driven Low-cost Fuzzy Controllers with Servo System Applications<br>Chairperson: Andreja Tepavčević |
| 10:00–11:00 | **Regular Session 2**<br>Chairperson: Vilém Novák |
| | *Optimizing Hierarchical Fuzzy Signatures for Masonry Compressive Strength Assessment*<br>András Kaszás, Vanda O. Pomezanski, László T. Kóczy |
| | *Applying The Fuzzy Signatures Concepts for Linguistic Values*<br>Nour Ammar, László T. Kóczy |
| | *A First Exploration of Concept Representation in Latent Space*<br>Arturo Fernández-Mora, Juan Moreno-García, David Muñoz-Valero, Luis Martínez |
| 11:00–11:30 | Coffee break |
| 11:30–13:30 | **Special Session: Recent trends in knowledge representation and modelling. Part 2**<br>Chairperson: David Lobo |
| | *Application of Intermediate Quantifiers Forming a Logical Structure of Opposition*<br>Vilém Novák, Petra Murinova, Karel Fiala |
| | *Real-valued semantics for the Multi-adjoint Gödel logic*<br>María Eugenia Cornejo, Francesc Esteva, Luis Fariñas-del Cerro, Lluis Godo, Jesús Medina |
| | *From Answer Set Programming to Epistemic Agents: A Hybrid Framework for Personalized Nutrition*<br>Alina Vozna, Stefania Costantini, Valentina Pitoni |
| | *Hybrid cd-implications to handle mixed information*<br>Fernando Chacón-Gómez, M. Eugenia Cornejo, Jesús Medina |
| | *Towards the non recursive notion of pseudo-intent in residuated concept lattices with hedges*<br>M. Eugenia Cornejo, Jesús Medina, Francisco J. Ocaña-Alcázar |
| | *A hybrid framework for integrating three-valued and fuzzy Formal Concept Analysis*<br>Francisco Pérez-Gámez, Pablo Cordero, Manuel Enciso, Ángel Mora |
| 13:30–15:30 | Lunch |
| 15:30–17:30 | **Timisoara tour** |
| 20:30 | **Gala dinner** |

| **WEDNESDAY 13th**<br>**Location: Continental Hotel** | |
|---|---|
| 10:00–10:50 | **Keynote Speaker - Annamária R. Várkonyi-Kóczy**<br>Title: Fuzzy Models in Anytime Systems<br>Chairperson: Manuel Ojeda-Aciego |
| 10:50–11:10 | Coffee break |
| 11:10–12:10 | **Regular Session 3**<br>Chairperson: Martin Kalina |
| | *Difference between divisibility and smoothness for pseudo-t-norms on finite lattices*<br>Juraj Kalafut, Martin Kalina, Ivona Hrivová, Andrea Mesiarová-Zeménková |
| | *Codomain lattice and optimization problems*<br>Vanja Stepanović, Andreja Tepavčević |
| | *Omega-algebras in classical fuzzy setting*<br>Vanja Stepanović, Andreja Tepavčević, Aleksandar Trkulja |
| 12:10–13:30 | **Regular Session 4**<br>Chairperson: Pablo Cordero |
| | *Fairness Distortion Index: A Fuzzy Aggregation Framework for Evaluating League Scheduling Bias*<br>Boldizsár Tüű-Szabó, László T. Kóczy |
| | *From Voting to Aggregation: An Aggregation-Based Interpretation of Alternative Voting Mechanisms in the Fuzzy Hough Transform*<br>Szilvia Nagy, Brigita Sziová |
| | *On aggregation of $T_M$-fuzzy vector subspaces*<br>Dragan Jočić, Ivana Štajner-Papuga |
| | *On discrete Sugeno integral and aggregation of algebraic objects*<br>Ivana Štajner-Papuga, Andreja Tepavčević |
| 13:30–15:50 | **Closing Session – Lunch** |

| Social Events | |
|---|---|
| **SUNDAY 10th** | |
| 08:30 | **Tour to Arad and Balla Geza Winehouse, Minis** |
| | |
| **MONDAY 11th** | |
| 20:00 | **Welcome reception** |
| | |
| **TUESDAY 12th** | |
| 15:30–17:30 | **Timisoara tour** |
| 20:30 | **Gala dinner** |
| | |
| **WEDNESDAY 13th** | |
| 13:30–15:30 | **Closing Session** |

***Keynote speech*:**

# Three-way decision: A computational intelligence perspective



**Yiyu Yao**

Department of Computer Science,
University of Regina, Canada.

**Abstract:** A theory of three-way decision is about thinking in threes, problem solving in threes, and computing in threes. This talk covers the principles, methods, and applications of three-way decision from a computational intelligence perspective. The three parts of the talk are (a) basics of three-way decision, (b) thinking in threes in computational intelligence and mathematics, and (c) implications of three-way decision to computational intelligence.

# A Personality and Emotional Disregulation-based Decision system for categorizing online gambling addiction levels

**Jesús Serrano-Guerrero[1], Woorod Al-Muhtaseb[1,2], Abdullah Abdulgafer[1,2], Guadalupe Plaza[1], Andrés Montoro-Montarroso[1], Francisco P. Romero[1] and José A. Olivas[1]**

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**Abstract:** Gambling disorder remains largely undetected due to a significant "help-seeking gap." This study proposes a scalable decision system for early detection by monitoring social media narratives. By leveraging Large Language Models and ontologies, the system identifies "digital fingerprints" of personality (UPPS-P) and emotional dysregulation-key factors where gambling serves as "emotional self-medication." The architecture transforms unstructured discourse into clinical indicators, integrating impulsivity facets and affective states to generate early warnings. This approach represents a vital step toward proactive, personalized prevention in behavioral addictions.

**Keywords:** Gambling disorder · Impulsivity · Emotional dysregulation · Large Language Models · Social media

**Acknowledgement:** This study has been supported by the Spanish Ministry of Social Rights, Consumption and Agenda 2030 under the project Detec-EMO 2.0 (SUBV24/00018).

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# Benchmarking Metaheuristic Algorithms for Revenue-Driven Timetabling in Deregulated Railway Markets

**David Muñoz-Valero[1], Juan Moreno-García[1], Julio Alberto López-Gómez[2] and Enrique Adrián Villarrubia-Martín[2]**

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**Abstract:** The liberalization of the European railway sector requires new approaches to coordinate the commercial interests of Railway Undertakings (RUs) with the capacity management objectives of Infrastructure Managers (IM) [1]. This contribution presents a summary of the work originally published in [2], which addresses the Train Timetabling Problem (TTP) in a deregulated market. The original study formulates the TTP as an optimization problem aimed at maximizing IM revenue while minimizing penalties derived from deviations in RUs' requested schedules. This work outlines the modular simulation framework presented in [2] to evaluate the performance of ten different metaheuristic algorithms—including Genetic Algorithms (GA), Particle Swarm Optimization (PSO), and Differential Evolution (DE)—implemented via the MEALPY library [3]. Furthermore, the scalability of these approaches is analyzed by comparing them against the SCIP exact mathematical solver [4] modeled in Pyomo [5]. Experimental results indicate that while exact methods struggle with scalability in large instances (e.g., 50 service requests), the Genetic Algorithm consistently delivers superior performance in terms of revenue maximization, convergence speed, and schedule adherence. The study concludes that properly tuned metaheuristics offer a robust and efficient solution for tactical planning in competitive railway environments.

**Keywords:** Railway Timetabling · Metaheuristics · Deregulated Market · Genetic Algorithm · Optimization

**Acknowledgement:** This work was supported by grants PID2020-112967GB-C32 and PID2020- 112967GB-C33 funded by MCIN/AEI/10.13039/501100011033, by ERDF A Way of Making Europe and the Research Vice-Rectory of the Universidad de Castilla-La Mancha. It was completed when Enrique Adrian

Villarrubia- Martin was a predoctoral fellow at the Universidad de Castilla-La Mancha funded by the European Social Fund Plus (ESF+).

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# Fairness-Aware Optimization for Train Timetabling in Liberalized Railway Markets: A Comparative Study of Equity Metrics

**David Muñoz-Valero[1], Juan Moreno-García[1], Julio Alberto López-Gómez[2] and Enrique Adrián Villarrubia-Martín[2]**

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**Abstract:** The liberalization of the European railway sector has introduced competition among Railway Undertakings (RUs) for limited infrastructure capacity, challenging Infrastructure Managers (IM) to allocate slots not only efficiently but also equitably. This contribution summarizes the research currently under review in [1], which proposes a fairness-oriented timetabling framework. Unlike traditional approaches that prioritize IM revenue—often leading to monopolistic behaviors—the proposed methodology integrates equity metrics such as the Jain index [2], Gini coefficient [3], and Atkinson index [4] into the optimization process. A Genetic Algorithm is employed, implemented via the MEALPY library [5] coupled with a conflict resolution heuristic designed to protect the most disadvantaged operators. Experimental results, simulated on a high-speed corridor using the ROBIN simulator [6], demonstrate that incorporating these metrics significantly reduces allocation inequity across different competitiveness scenarios. The study highlights a quantifiable trade-off between maximizing revenue and ensuring fair market access, supporting the use of these equity indices as decision-support tools for sustainable railway ecosystems.

**Keywords:** Railway Timetabling · Fairness · Liberalized Market · Equity Metrics · Genetic Algorithm.

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# Deconvolutional Networks as an XAI Method for Convolutional Neural Networks in Brain MRI Classification

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**Abstract:** In medical applications, achieving high predictive performance is necessary but not sufficient; the rationale behind a model's decisions must also be assessable. Convolutional neural networks (CNNs) can deliver strong results in magnetic resonance imaging (MRI) analysis, yet their decision-making process is difficult to interpret. This article presents deconvolutional networks as a local explanation method for individual CNN predictions. The core mechanism is reviewed, including the backward projection of activations from feature space to the input domain via unpooling and transposed convolutions. The method is discussed in a four-class brain MRI classification setting, comparing a shallow custom CNN with a transfer-learned VGG19 model. Numerical performance (94.69% and 97.73% test accuracy, respectively) is complemented by qualitative observations derived from deconvolutional visualisations and their relation to perturbation-based explanations. The analysis highlights practical benefits of deconvolutional views for sanity-checking spatial focus, as well as limitations in multi-class settings and in deeper reconstructions for transfer-learned models.

**Keywords:** deconvolutional networks · explainable AI (XAI) · activation map visualisation · transposed convolution · unpooling · CNN · brain MRI analysis.

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# A Scenario-Based Fuzzy Cognitive Map Approach to ESG Analysis in Higher Education

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**Abstract:** This paper presents a scenario-based modeling framework using fuzzy cognitive maps (FCMs) for analyzing the interactions among environmental, social, and governance (ESG) dimensions in higher education systems. In the proposed model, key ESG areas are represented as interconnected concepts whose relationships are described through a weighted cognitive map. The dynamic behavior of the system is examined through an iterative update mechanism combined with a sigmoid activation function, allowing causal influences to propagate across the network.

Scenario analysis is conducted by applying moderate perturbations to selected ESG components and simulating the resulting system dynamics. The simulations reveal that the modeled system consistently converges to a stable equilibrium state across all examined scenarios. Although targeted interventions produce measurable short-term deviations during the transient phase, the long-term system behavior remains unchanged.

These results indicate the presence of a dominant attractor in the modeled system and suggest structural stability with respect to moderate perturbations. The study highlights the usefulness of fuzzy cognitive maps as an exploratory modeling and decision-support tool for analyzing complex interdependencies and dynamic responses in ESG-oriented higher education systems.

**Keywords:** ESG assessment · higher education · fuzzy cognitive maps · scenario analysis · system dynamics· sustainability

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# Fuzzy Cognitive Map Model reduction with Neural Network

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**Abstract:** This paper presents a behavior-preserving reduction method for Fuzzy Cognitive Maps (FCMs) based on a modified Multilayer Perceptron (MLP). Instead of learning static input-output mappings, the proposed approach focuses on preserving the dynamic behavior of the original system during the reduction process. The quality of the reduction is evaluated by comparing output concept trajectories over multiple update steps using the Mean Absolute Error (MAE). The results show that the essential dynamics of the original FCM can be preserved across different reduction levels with low computational cost.

**Keywords:** Fuzzy Cognitive Maps · Reduction · Multilayer Perceptron

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***Keynote speech*:**

# Harnessing AI with Limited Data: Approximate Knowledge Interpolation and Practical Applications

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**Abstract:** AI stands to transform nearly every aspect of contemporary life. Much of its success is driven by deep learning techniques that rely on vast quantities of data. Yet, a pivotal question emerges when faced with limited data for a new problem, especially if such data is ambiguously characterised. Can AI maintain its efficacy under these constraints? This talk delves into contributions addressing this query, highlighting how fuzzy rule interpolation (FRI) enables approximate reasoning in situations marked by sparse or incomplete knowledge. This is particularly relevant when traditional rule-based inference mechanisms falter because observations do not align with existing rules. The talk will centre on a prominent subset of FRI techniques, Transformation-based FRI (T-FRI). Kicking off with an exploration of the foundational T-FRI approach, it will segue into a concise overview of its expanded repertoire, each addressing certain shortcomings inherent to the original method. Subsequently, real-world applications of these methodologies will be showcased, exemplifying their potency in tackling formidable challenges in domains like network security and medical diagnosis. These cases will underscore AI's capability to function effectively even with incomplete knowledge and ambiguous data. The talk will conclude with a brief look at promising future directions in this vital area of research.

# From fuzzy measures to communities: a clustering approach for interpretation

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**Abstract:** In this paper, we address the challenge of interpreting fuzzy measures. The goal is to obtain a partition of the set of agents over which the fuzzy measure is defined, providing useful information about their relevance, interactions and contributions to coalitions. To this end, we use a network-based visualization of the agents and their relationships, obtained with the Shapley value and the interaction index. We then propose an aggregation methodology that summarizes the relevant information of a fuzzy measure into a matrix, to which we apply the Louvain algorithm in order to obtain partitions of the agents. With an appropriate representation, this network partition provides useful, clear, and immediate information about the synergies defined by the fuzzy measures over the agents.

**Keywords:** Fuzzy measures · Networks · Clustering · Shapley value · Interaction Index · Louvain algorithm

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# Formal Concept Analysis for Evaluating the Intent Dimension of LLMs

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**Abstract:** We propose a framework based on Formal Concept Analysis (FCA) to audit the "intent dimension" of LLMs. By operationalizing the model as an oracle in Attribute Exploration, we introduce an entropy-based dynamic retry mechanism to mitigate stochasticity. Experiments across models (1.5B–120B) in Integer and Graph domains reveal a "Generative Gap": models often validate abstract rules but fail to generate valid counterexamples. This structural inconsistency suggests LLMs rely on statistical correlation rather than stable conceptual grounding.

**Keywords:** Formal Concept Analysis · Language Models · Concept Exploration

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# On the aggregation of contexts with common attributes

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**Abstract:** The aggregation of databases enables the local study of decentralized and partially overlapping information, and formal concept analysis provides a rigorous mathematical framework for addressing this task. This paper investigates the aggregation of databases with common attributes in formal concept analysis, focusing on how information is extended when passing from local data sources to a unified global system.

**Keywords:** Formal concept analysis · context aggregation · concept lattice

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# Interactive Fuzzy FCA Recommender System: A Visual Approach using fcaR

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**Abstract:** Traditional recommender systems often operate as black boxes, making them difficult to integrate into critical domains such as medical diagnosis, where explainability is mandatory. This paper introduces an interactive recommendation framework based on the navigation of Fuzzy Formal Concept Lattices. Rather than relying on offline calculations or static rule sets, our approach implements a *Guided Recommender* Algorithm that allows experts to explore the knowledge structure iteratively. By applying a *Local Zoom Strategy*, the system extracts relevant sublattices in real-time, enabling users to refine searches through attributes with fuzzy degrees of certainty.

We provide a tool called fcaRvizFuzzy, developed in R/Shiny, which will be integrated with the fcaR package in the medium term. We demonstrate the utility of the tool using the cobre32 schizophrenia dataset, showing how a step-by-step visual refinement process facilitates rigorous decision-making support for individuals without mathematical expertise.

**Keywords:** Fuzzy Formal Concept Analysis · Recommender Systems · Explainable AI (XAI) · fcaR · Medical Diagnosis · Visualization

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# Definition and resolution of bipolar multi-adjoint relation equations

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**Abstract:** Recent literature has addressed bipolar fuzzy relation equations with join-irreducible right-hand side within the context of complete distributive residuated lattices. This work contributes to the advancement of the study by transitioning to the multi-adjoint paradigm, which offers a more flexible framework that weakens traditional algebraic constraints and replaces standard triangular norms with conjunctions of adjoint pairs. The present study focuses on the resolution of resolution of bipolar multi-adjoint relation equations with join-irreducible right-hand side, paying particular attention to their solution set.

**Keywords:** Bipolar multi-adjoint relation equation · distributive symmetric right unital multi-adjoint lattice · negation operator · join-irreducible element

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# Characterizing the solvability of bipolar sup-equations in terms of the negation operator

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**Abstract:** Bipolar sup-equations are the building stones of bipolar fuzzy relation equations, which are a modified version of fuzzy relation equations where a negation operator is considered. The resolution of bipolar sup-equations, and therefore of bipolar fuzzy relation equations, is directly related to the relation sup-composition operator and the negation operator under consideration.

This work will focus on characterizing the solvability of bipolar sup-equations on the unit interval by imposing conditions exclusively on the negation operator. The fundamental objective is to generalize already known characterizations, which require conditions on both the sup-composition and the negation operator involved in a bipolar sup-equation.

**Keywords:** Bipolar sup-equation · bipolar fuzzy relation equation · negation operator · unit interval · solvability

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***Keynote speech*:**

# Model-based and Data-driven Low-cost Fuzzy Controllers with Servo System Applications

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**Abstract:** This keynote talk covers the following topics: an overview of the Process Control Group at the Politehnica University of Timișoara in Romania; data-driven versus model-free control; our contributions; the structure and tuning of two-degree-of-freedom (2-DOF) fuzzy controllers; servo system control applications; and evolving fuzzy systems in relation to control. The presentation will include transportation applications within the context of an ERA-NET Cofund Urban Accessibility and Connectivity (ENUAC) JPI Urban Europe project.

# Optimizing Hierarchical Fuzzy Signatures for Masonry Compressive Strength Assessment

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**Abstract:** Non-destructive testing methods are widely used in civil engineering. Such are estimating masonry compressive strength from in situ tests. In this paper, a hierarchical fuzzy signature model is presented, that combines visual observations with non-destructive measurements. In our previous research optimized weighted geometric mean was used for all internal aggregations, in accordance with preliminary expert proposed evaluation methods. In this research the goal was to determine more fitting aggregations by using evolutionary algorithm to determine the optimal linear combination of weighted geometric mean and weighted exponential mean operators, with all associated aggregation parameters, which were tuned so that the fuzzy-signature root output best agrees with reference compressive strengths obtained from destructive laboratory testing. To optimize the parameters of the model, two metaheuristic procedures were applied, namely, genetic algorithm and particle swarm optimization. Experiments on a real dataset showed that the optimization procedures can rather well approximate even extreme objective-function values, and validation with real measurements confirmed that the resulting model can support practical structural assessment.

**Keywords:** Fuzzy signature · Weighted geometric mean aggregation · Weighted exponential mean aggregation · Masonry strength · Evolutionary optimization of general aggregation.

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# Applying The Fuzzy Signatures Concepts for Linguistic Values

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**Abstract:** Fuzzy linguistic approaches are widely used in decision-making problems that rely on qualitative reasoning and human-like assessment. Their strength lies in modeling uncertainty and subjectivity through linguistic labels such as low, medium, high, etc. However, classical fuzzy linguistic models and fuzzy rule-based systems usually employ a flat representation, where linguistic variables are treated as independent attributes. This flat structure limits the ability to capture inter-variable dependencies, hierarchical relationships, and natural groupings that commonly exist in real-world problems. As a consequence, aggregation in flat models often relies on approximation, leading to information loss, reduced expressiveness, and diminished precision, especially as system complexity increases. To address these structural limitations, this paper proposes the use of Fuzzy Signatures (FSigs) as a hierarchical framework for linguistic information representation and aggregation. Fuzzy signatures enable linguistic variables to be organized into nested structures that reflect domain relationships and preserve intermediate reasoning results. By supporting flexible aggregation at multiple levels, fuzzy signatures reduce information loss, improve interpretability, and allow more accurate modeling of complex decision-making problems. The applicability of the proposed approach is illustrated through simple intelligent traffic control scenario.

**Keywords:** Fuzzy signatures · Linguistic variables · Linguistic Aggregation

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# A First Exploration of Concept Representation in Latent Space

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**Abstract:** Autoencoders are models designed to learn compressed representations of data while preserving its most essential information. During the encoding process, they generate a latent space—an internal structure in which the complexity of the original data is transformed into a more compact and abstract representation. This naturally raises the question of whether the latent space captures the patterns the model has discovered in an unsupervised manner. In parallel, activation vectors provide a record of the response of individual neurons to each input, offering a complementary perspective for analysing internal representations. For these reasons, the way in which the latent space encodes conceptual information is examined by analysing activation vectors through clustering techniques, with the aim of determining whether the organisation of learned knowledge can be detected. The goal is to relate the resulting clusters to the concepts represented in the inputs. Our initial experiments focus on identifying conceptual patterns in activation vectors derived from simple and easily interpretable images, which facilitate concept detection. Once the experiments are completed, the results will be analysed and an attempt will be made to relate the emerging clusters to the underlying concepts.

In [1] an initial approach to this problem by exploring explainability in deep neural networks through the extraction of concepts learned in intermediate layers has been detailed. Autoencoders were used to analyse latent representations that model the input, filter noise, and capture relevant patterns that help identify significant features [2]. The study is divided into two complementary parts. First, an autoencoder was incorporated into an architecture designed for text polarity classification. Second, the DistilROBERTa model [3] was used to identify the dominant emotion present in each input text. Using both the extracted emotions and the neural activations, several clustering algorithms were applied to examine the conceptual organisation of the data. Cluster quality was assessed through internal cohesion and intercluster separation metrics based on

cosine similarity [4,5]. Although the results did not reveal a clear correspondence between clusters and emotions, certain groups exhibited high internal cohesion—particularly those generated with Fuzzy C-Means and Agglomerative Clustering—suggesting that clustering may still capture meaningful patterns in the latent space. These findings highlight the inherent complexity and subjectivity of emotion analysis in text, making its latent space difficult to interpret. Consequently, we propose to analyse concepts in a collection of synthetic images featuring clear and non-subjective concepts.

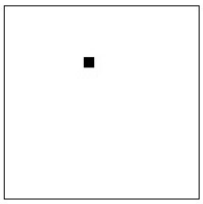

**Fig. 1.** An example of an input figure

The present experiment investigates how the position of a square within an image is represented in the latent space (Figure 1). The aim is to analyse how the model organises information within neurons and activation vectors to encode the square's central coordinates. We seek to obtain clusters that exhibit strong internal coherence and clear separability. Given that the represented concepts are entirely objective, this experiment is expected to yield more interpretable results and improve upon the findings of our initial approach.

**Keywords:** interpretability · deep learning · machine learning · activation vectors · concepts

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# Application of Intermediate Quantifiers Forming a Logical Structure of Opposition

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**Abstract:** In this paper we will present application of fuzzy natural logic to the analysis of scientific data and their description using the, so called, evaluative linguistic expressions and intermediate quantifiers. We show that their theory can be applied to data with the goal to produce summarisation of data in linguistic form.

**Keywords:** Fuzzy natural logic · Intermediate quantifiers · Intermediate syllogisms · Graded cube of opposition · Graded hexagon of opposition · Evaluative linguistic expressions

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# Standard completeness of multi-adjoint Gödel logic

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**Abstract:** Multi-adjoint Gödel logic ($\mathrm{MGL}_{\Delta}$) is an algebraizable variant of the Multi-adjoint Lattice logic that was shown to be complete with respect to a corresponding class of linearly-ordered algebras. In this paper, we strengthen this completeness result in the sense that we show that $\mathrm{MGL}_{\Delta}$ is also standard complete, that is, it is complete with respect to the proper subclass of algebras defined on the real-unit interval.

**Keywords:** Multi-adjoint lattice logic · Gödel Logic · Baaz Monteiro projection · Completeness

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# From ASP to L-DINF: Towards Epistemic Dietary Agents for Personalized Nutrition

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**Abstract:** Personalized nutrition systems balance strict constraints (e.g., allergies) and dynamic user adaptation. While Answer Set Programming (ASP) offers robust optimization, it lacks the cognitive flexibility for real-time belief revision. We introduce a hybrid dietary framework integrating ASP optimization with L-DINF, an epistemic logic for cognitive agents. This approach translates static "Blueprint Personas" into dynamic agents capable of introspection and negotiation by reasoning about beliefs, intentions, and the feasibility of actions. Implemented in the DALI agent language, this integration achieves the adaptive, explainable decision-making critical for modern dietary coaching.

**Keywords:** Answer Set Programming · Blueprint Personas · Epistemic Logic · Cognitive Agents · Personalized Nutrition.

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# Hybrid cd-implications to handle mixed information

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**Abstract:** Mixed attribute implications have been introduced in Formal Concept Analysis to model positive and negative information simultaneously. This paper presents an alternative approach based on hybrid conjunctive-disjunctive attribute implications, which do not require the explicit inclusion of negative attributes to capture mixed information. In addition to considering fewer attributes, these hybrid implications also enable more comprehensive knowledge extraction due to their more flexible structure.

**Keywords:** Formal Concept Analysis · Mixed Information · Attribute Implications · Knowledge Extraction

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# Towards the non recursive notion of pseudo-intent in residuated concept lattices with hedges

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**Abstract:** Pseudo-intents play an important role in the computation of attribute implication bases in residuated concept lattices with hedges. In this paper, we propose a non-recursive notion of pseudo-intent for the residuated framework with the aim of providing deeper conceptual insight and clarifying its structural properties. We study the relationship between this new notion and the previously established recursive definition, identifying conditions under which the two notions are equivalent.

**Keywords:** Pseudo-intent · residuated concept lattice · truth-stressing hedge · attribute implication.

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# A hybrid framework for integrating three-valued and fuzzy Formal Concept Analysis

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**Abstract:** Formal Concept Analysis (FCA) is a fundamental tool for extracting knowledge from data organized in a formal context. Its theoretical foundation stems from the notion of Galois connections and generalizations of Birkhoff's theorem. The extracted knowledge can be expressed in two ways: as a set of attribute implications (a theory) or as a formal concept lattice (a hierarchy of concepts). Since its inception [1], several extensions of FCA theory have been developed to model different types of information within a formal context. Such extensions require generalizing the notion of formal context and the underlying Galois connection on which the machinery relies. Furthermore, this process necessitates a broader generalization of both attribute implications and formal concepts. Our research focuses primarily on how missing information is treated and interpreted, highlighting the strong relationship between this issue and the semantics of negation.

One of the early extensions in this area [2] proposed that if an object cannot be affirmed to possess an attribute, the negation of that affirmation is assumed. While this initial approach lacked the expressive power to capture the complexity of real-world data, it provided a necessary foundation for further study. Consequently, in [3], we explored this framework and developed a sound and complete axiomatic system where missing information is treated as negation.

A second step, more suitable for real-world applications, involves distinguishing between negated and unknown information. In [4], we introduced a three-valued framework to capture all possibilities: affirmed, negated, or unknown. Although various approaches incorporate three-valued logic—often integrated into different frameworks and analyzed in detail in [5]—our approach [4] adopts an ordering of truth values induced by the amount of knowledge, similar to the bilattice approach [6]. Additionally, we defined several sound and complete axiomatic systems with distinct orientations.

Furthermore, a prominent line of FCA extensions is framed within fuzzy set theory, with numerous works developing fuzzy generalizations [7,8,9]. The integration of these fuzzy frameworks with the aforementioned three-valued perspective leads to the consideration of interval-valued or intuitionistic logic. While initial works exist in this area [10,11], both semantics differ and they are not equivalent. In this work, we propose an extension of FCA rooted in our previ-

ous research [4], blending it with other approaches within the intuitionistic or interval-valued paradigms.

**Keywords:** Formal Concept Analysis, unknown information, Intuitionistic

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***Keynote speech*:**

# Fuzzy Models in Anytime Systems

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**Abstract:** In resource, data, and time insufficient conditions, anytime algorithms, models, and systems can be used advantageously. Anytime systems are real-time systems with the aim to provide continuous operation in case of changing circumstances and to avoid critical breakdowns in cases of missing input data, temporary shortage of time, or computational power. Naturally, while the information processing can be maintained, the complexity must be reduced, thus the results of the computing become less accurate. Although, it is ensured that the results are the best in the given circumstances and that the error of the output is always known. Embedding fuzzy (and neural network) models in anytime systems extends the advantages of anytime systems, e.g. with respect to the transient behavior of the dynamic systems. Unfortunately, however, in many cases we face difficulties finding appropriate fuzzy models to be adopted. The talk focuses on different methods and techniques that can be applied to transform 'classical' fuzzy models into appropriate models for easy and effective anytime use. The concepts and techniques of anytime systems together with solutions of anytime fuzzy (and neural) approaches will be addressed. To illustrate efficiency, in the presentation, several applications will also be discussed, focusing on areas such as signal-, image processing, fault diagnosis, and control.

# Difference between divisibility and smoothness for pseudo-t-norms on finite lattices

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**Abstract:** Continuous t-norms are undoubtedly the most studied t-norms. However, continuity is not a suitable notion for finite lattices. Therefore, smoothness, divisibility, 1-Lipschitzianity, and the intermediate value property were proposed as suitable counterparts of continuity.

This paper is devoted to studying these properties of t-norms on finite lattices. These three properties coincide only on some special types of lattices. Smoothness and divisibility are shown to be independent properties of t-norms, in general. Furthermore, on finite lattices with incomparable elements, divisibility and smoothness of pseudo-t-norms do not necessarily imply commutativity, which differs from the situation of pseudo-t-norms on finite chains.

**Keywords:** T-norm · finite lattice · divisibility · smoothness · pseudo-t-norm · intermediate value property

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# Codomain lattice and optimization problems

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**Abstract:** The paper gives a survey of the existing knowledge about optimal solutions to lattice-valued relational equations and inequations, adding some new results to complete what is still lacking. A necessary and sufficient condition for having the greatest solution of the typical inequation is the infinite distributivity of the codomain lattice. The least solution, however, need not exist even in the crisp case. Meet-continuity of the codomain lattice is a sufficient condition for the existence of a maximal solution to any system of relational equations and inequations. A maximal solution may not exist without meet-continuity, even if the codomain lattice is distributive. However, we give an example proving that - in case of a finite domain set - meet-continuity is not necessary condition for having a maximal solution to all typical inequations. A minimal solution does not exist in the case of a residual codomain lattice. It may not exist even in a more special case of complete Heyting codomain lattice.

**Keywords:** Lattice-valued relational (in)equation · The greatest solution · The least solution · A maximal solution · A minimal solution

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# Omega-algebras in classical fuzzy setting

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**Abstract:** An $\Omega$-algebra is an algebra paired with a lattice-valued equality, i.e., with a symmetric and transitive fuzzy relation, compatible with all the algebra operations. Its cuts are weak congruence relations of the initial crisp algebra and vice versa; if in an $\Omega$-algebra all the cuts are weak congruences, it is a fuzzy equality. In this paper, we take the classical fuzzy relations and get some specific properties: the cuts of the fuzzy weak-congruence relations of an $\Omega$-algebra form complete chains in the weak congruence lattice of the initial crisp algebra. Starting from this, we prove that any complete finite or countable chain-sublattice of the weak congruence lattice of an algebra can be represented as a chain of cuts of an $\Omega$-algebra with the same crisp algebra. We illustrate the property we prove with two examples of chains in the weak congruence lattices of algebras, represented with the family of cuts of $\Omega$-groups.

**Keywords:** $\Omega$-algebra · fuzzy equality · weak congruences

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# Fairness Distortion Index: A Fuzzy Aggregation Framework for Evaluating League Scheduling Bias

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**Abstract:** The structure of a league schedule can influence competitive outcomes independently of team strength, raising concerns about procedural fairness. Previous research introduced a Monte Carlo–based framework for measuring schedule-induced distortions using two indicators: the Total Fairness Bias (TFB), quantifying the overall deviation of final standings from an unbiased reference, and the Gini index, expressing how unevenly this distortion is distributed among teams. Although this two-dimensional description offers valuable insight, it does not provide a single value that supports straightforward comparisons between leagues, seasons, or alternative scheduling procedures. Moreover, identical numerical values of TFB or Gini may represent different fairness levels across competitions with distinct structural properties. To address these limitations, this paper proposes the Fairness Distortion Index (FDI), a Mamdani-type fuzzy inference system that aggregates TFB and Gini into a unified scalar measure in the interval [0,1]. The FDI distinguishes between ethically different distortion patterns – most notably, small global distortions that disadvantage only a few teams versus systemic distortions that affect all participants equally. As such, the FDI yields an interpretable and context-sensitive measure of unfairness. Beyond retrospective evaluation, it can serve as an optimisation objective in schedule generation, enabling the search for fixture lists that minimise competitive bias under existing competition formats.

**Keywords:** Fuzzy systems · Fairness measurement · Sports competitions

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# From Voting to Aggregation: An Aggregation-Based Interpretation of Alternative Voting Mechanisms in the Fuzzy Hough Transform

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**Abstract:** The Hough transform and its fuzzy extensions are traditionally formulated as voting-based procedures in a discretized parameter space. While numerous alternative voting mechanisms have been proposed to improve robustness and reduce the influence of noise, their relation to aggregation theory has received limited explicit attention. In this paper, we reinterpret voting in the linear fuzzy Hough transform as an aggregation process that fuses local fuzzy evidence into global line hypotheses. We show that classical, fuzzy, and alternative weighted voting schemes can be viewed as specific instances of aggregation operators applied to per-point fuzzy contributions in the accumulator space. This aggregation-based viewpoint provides a unifying framework for analyzing robustness, peak concentration, and suppression of spurious maxima. The proposed interpretation is illustrated using a synthetic benchmark and the analysis of grid structures in electrocardiogram (ECG) images, where dense and repetitive line patterns can yield many local maxima under naive voting. Placeholder experiments indicate that aggregationdriven voting can improve dominant orientation estimation and reduce spurious peaks in highly structured scenes. The framework is general and can be extended to other parametric curves and application domains.

**Keywords:** Fuzzy Hough transform · aggregation functions · voting mechanisms · line detection · ECG image analysis

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# On aggregation of $T_M$-fuzzy vector subspaces

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**Abstract:** Aggregation operators, as mathematical tools, play an important role in fusing data within many areas and disciplines. When the data to be aggregated have a specific structure, it is particular interest to know under which condition this structure is preserved under aggregation. In this paper focus is on n-ary aggregation of special classes of fuzzy vector subspaces on sets. The obtained results extend previously obtained ones for binary aggregation of $T_M$-fuzzy vector subspaces on sets.

**Keywords:** aggregation operators · t-norms · fuzzy sets · $T_M$-fuzzy vector subspaces

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# On discrete Sugeno integral and aggregation of algebraic objects

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**Abstract:** It is well known that, algebraically speaking, the family of all substructures of a structure is closed under intersection. However, the union, that can be viewed as a form of aggregation, preserve the given structure only in certain cases, e.g., the union of two subgroups is a subgroup if and only if one of them is contained in the other one. Similar to the classical setting, aggregation of two fuzzy subgroups does not have to result in a fuzzy subgroup. A few specific situations with a positive result are described in [1,2,9], however, the aggregation processes of specific type of lattice-valued algebraic structures remains a field of interest for the further investigation, specially directed towards studies on the types of aggregation operators that are convenient for different uses of empirical data. Since the approach is lattice-valued, the assumption is that the aggregation operators are compatible with the lattice structure ([4]). Therefore, the focus of this research is on discrete Sugeno integral as an aggregation operator, as well as on $\Omega$-groups, where $\Omega$ is a complete distributive lattice ([7]), and fuzzy groups as input structures.

Some recent results concerning aggregation of $\Omega$-groups can be found in [8]. Since a function $f\colon [0,1]^n \to [0,1]$ is an $n$-ary Sugeno integral if and only if it is an idempotent weighted linear polynomial function (see [5]), conclusions from [8] can be applied to discrete Sugeno integrals. Since the Sugeno integral is widely recognized as a qualitative aggregation operator whose lattice-based form is well suited for combining heterogeneous and ordinal data, and since the previous observation provides a natural connection between the abstract algebraic framework and this well-known qualitative aggregation tools, further investigation of Sugeno-type aggregation operators is more than justified.

The main aim of this paper is to extend results form [8] to Sugeno-type integrals as aggregation operators within the proposed $\Omega$-group framework. Additionally, since the lattice-valued formulation allows the aggregation process to preserve the inherent imprecision of data while supporting flexible notions of equivalence through the aggregation of weak congruences, possible application in digital forensics [3,6] is being considered.

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